Analytical modeling of the generation of waves by the wind according to Jeffrey’s theory in deep water

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ABSTRACT

The swell is one of the main causes of degradation ribs and one of the factors that makes navigation difficult when it tends to be extreme. Wind is one of the main causes of the birth of the waves. In this work it was considered as the main factor, we proposed an analytical prediction model based on the Weibull distribution and shelter Jeffrey’s mechanism. Using the condition maturity of a wave got work Wu (1982) and Charnock (1955), the results obtained show that the speed, the period and wavelength of an emerging wave are only increasing functions of wind speed and time. As for the height of the wave, it depend not only these parameters but also the fetch. The period after which a wave becomes an adult is different from the length of fetch.

Keywords: Wind; Wave of birth; swell; wave height, speed of waves, time and wavelength.

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\[ \rho = \frac{E}{\nu} \] Wave age
\[ \tau : \] The wave of growth period.

INTRODUCTION

Africa is a "hot spot" of climate change, particularly because of its vulnerability. The improved climate information (CI) and development an Early Warning System (EWS) are an effective way to help the awareness of the population against the risks climate so that they prepare for the consequences. Countries in sub-Saharan Africa are exposed to disasters including extreme drought late and heavy rains, foods, excessive heat and strong winds. The damages caused are enormous for development, while the mechanisms Prevention of these risks are almost nonexistent.

With observations over several years, researchers have shown the important role of Eastern Atlantic Ocean
in the onset and intensity of the West African monsoon.

We are interested in generation of waves by wind in deep water. This work aims to propose an analytical model to predict characteristics wave knowing those wind. The birth of the waves on the surface of the ocean is due to energy transfers between wind and water (B. Franklin, 1774). The waves experiencing a growth phase that continues as long as their speed remains below that of the wind. This growth is determined by three factors: the fetch, wind speed and gravity.

We recognize that wind and wave born, propagate in the same direction; the speed of the wave is growing greatness to its age adult. In this work, we modeled the evolution of a wave with the Jeffrey’s equation modified, the Weibull distribution and condition of maturity of a wave proposed by Wu (1982) and Charnock (1955). The results help to predict the behavior of a wave after his birth, knowing the characteristics of the wind.

**METHODS**

**Jeffrey’s Equation modified**

Jeffreys (1925) showed that the pressure fluctuations which acting on the free surface of the ocean at rest due to the flow of air, producing an energy transfer from the wind to the waves such per unit area: (CHAMBARREL Julien; Montalvo, 2013, p3108)

$$\frac{dE}{dt} = \int_{S_{atm}} p u dS$$

(1)

Having assumed that this transfer is solely due to form drag associated with delimitation occurring adjacent crests (face leeward) it showed that the atmospheric pressure is given by: (Montalvo, 2013)

$$P = \frac{\rho u^2}{2} (U - C)^2 \frac{g}{c}$$

with \(\eta = \cos (kx - \omega t)\)

(2)

Total wave energy per unit of surface is:

$$E = \frac{1}{2} \rho g H^2 = \frac{1}{2} \rho g h^2$$

(3)

Substituting the expressions (2) and (3) in the Jeffreys equation modified, one obtains:

$$\frac{dE}{dt} = \frac{1}{2} \rho g H^2$$

(4)

The energy transfer causes exponential growth parameters waves emerging adulthood. The wave becomes an adult when \(C = \beta_0 U\). Let \(\tau_p = \frac{L}{u}\) is the length of fetch and \(\tau\) the time after which the wave continues to grow. Once the wave becomes adult, all settings continue to grow and remain constant \(U = C(\xi) = \beta(\xi) U\) with \(C(\xi) = \beta_0 U\). Thus, \(\beta(0) = 0\) and \(\beta(\xi) = \beta_0\).

When wind velocity \(U > 7.08 \text{km/h}\) hits the free surface of the ocean, it induces the birth of a wave. The report \(\beta = \frac{\xi}{u}\) allows characterizing the age of a wave (Polnikov, 2004).

The relationship established by Wu (1982) and Charnock (1955) allows defining the condition of maturity of a wave that is a swell: (Montalvo, 2013)

$$C_{20} = (0.8 + 0.016V) 10^{-4} = \frac{\rho \cdot 9.8 \cdot \xi}{\rho u^2} = \frac{\rho \cdot 9.8 \cdot \xi}{\rho u^2}$$

(5)

where \(u = U - C\) is the wind speed on the friction surface.

If \(\beta_0 = \frac{L}{u}\) is the adulthood of the wave, we have:

$$C_{20} = U - \sqrt{\frac{0.8 \cdot 0.016 \cdot V \cdot 10^{-4}}{0.0661}}$$

then \(\beta_0 = \frac{\xi}{u} = 1 - \sqrt{\frac{0.8 \cdot 0.016 \cdot V \cdot 10^{-4}}{0.0661}}\)

(6)

- \(\beta < \beta_0\) or \(C < \beta_0 U\), the wave is young and all parameters (wavelength, peak to valley height, period and speed) grow exponentially with time.
- \(\beta = \beta_0\) or \(C = \beta_0 U\), the wave becomes an adult (swell) and all its parameters \((T, H, L, C)\) are constants reach in time.
- \(\beta > \beta_0\) or \(C > \beta_0 U\), adult wave (waves) sweeping.

The dimensionless number that characterizes the fetch \(\beta_0 = \frac{L}{u}\) (fetch dimensionless) (Polnikov, 2004) and how long the free surface of the ocean was hit by the wind velocity \(U\) is almost constant \(\tau_p = \frac{L}{u}\). Referring to the Weibull distribution (Houekpohoe et al., 2014; Faida, 2010)

$$P(\xi) = K_0 \left(\frac{\xi}{\xi_p}\right)^{n-1} e^{-\left(\frac{\xi}{\xi_p}\right)^n}$$

(7)

with \(\nu = 1\) for the exponentially, we set:

$$\beta = \begin{cases} 
1 - e^{-\frac{\xi}{\xi_p}} & \text{if } 0 \leq t \leq \tau \text{ with } \xi = \beta_0 U \\
\beta_0 & \text{if } t > \tau 
\end{cases}$$

(8)

The expression of the phase velocity is:

$$c = \begin{cases} 
\left(1 - e^{-\frac{\xi}{\xi_p}}\right) & \text{if } 0 \leq t \leq \tau \\
\beta_0 U & \text{if } t > \tau 
\end{cases}$$

(9)

Period \(T\) and wavelength \(\lambda\) of the young wave function of time until adulthood are respectively:

$$T = \frac{2\pi}{\omega} = \begin{cases} 
\frac{2\pi u}{\xi} \left(1 - e^{-\frac{\xi}{\xi_p}}\right) & \text{if } 0 \leq t \leq \tau \\
\frac{2\pi \beta_0 U}{\xi} & \text{if } t > \tau 
\end{cases}$$

(10)
Figure 1a. Variations of phase velocity according to time and wind speed for \( F = 10 \text{ km} \)

Figure 1b. Variations of phase velocity according to time and fetch for \( U = 20 \text{ m/s} \)

\[
L = Cl = \begin{cases} \frac{\sqrt{\frac{g}{u}}}{2} & \text{if } 0 \leq t \leq \tau \\ \frac{\sqrt{\frac{g}{u}}}{2} & \text{if } t \geq \tau \end{cases}
\]

and \( K^2C = \begin{cases} \frac{g}{u^2} & \text{if } 0 \leq t \leq \tau \\ \frac{g}{u^2} & \text{if } t \geq \tau \end{cases} \quad (12)\]

Replacing the previous expressions in equation (4) we obtain:

\[
\frac{dH}{dt} - \frac{g}{u^2}H \left( 1 - e^{-\frac{u}{g}} \right) = 0
\]

The general solution of this equation is of the form:

\[
H(t) = A_0 e^{-\left( \frac{g}{u^2} \right) \left( 1 - e^{-\frac{u}{g}} \right) t}
\]

In deep waters, according to (Rosales Sierra, 2004), the wave height (adult wave) based on fetch \( F \) and velocity \( U \) of the wind near the sea surface is:

Jonswap: \( H_s = 0.00178 \frac{u^2}{g} \left( \frac{F}{u} \right)^{0.5} \) \quad (15)

The boundary conditions of \( H(t) \) is such that \( H(0) = 0 \) and \( H(\tau) = H_s \) with \( H_s \) the height of the adult wave given by JONSWAP.

\[
R(t) = H_s e^{-\left( \frac{g}{u^2} \right) \left( 1 - e^{-\frac{u}{g}} \right) t} \quad (16)
\]

with \( \tau = \frac{\tau}{u} \ln \left( \frac{\frac{g}{u^2}}{\frac{g}{u^2} - 1} \right) \)

**RESULTS**

Figure 1a shows changing in the speed of the waves with time and speed wind fetch constant while Figure 1b reflects changes in the speed depending on the time and fetch but constant wind speed.

The variation of the period based on the time and fetch speed Constant wind is described in Figure 2a. Figure 2b shows the evolution of the period with time and fetch at constant speed wind.

The evolution of the wavelength depending time constant fetch for different values of speed Wind is shown in Figure 3a. Figure 3b describes the variation of the wavelength with time at constant wind speed and different value fetch.
Figure 2a: Variations of wave period according to time and wind speed for $F = 10 \text{ km}$.

Figure 2b: Variations of wave period according to time and fetch for $U = 20 \text{ m/s}$.

Figure 3a: Variations of wavelength according to time and wind speed for $F = 10 \text{ km}$. 
With Figure 4a we observe the evolution of the wave height based time constant fetch for different values of speed wind. Figure 4b shows the evolution of height versus time at constant wind speed and different value fetch.

Figure 5 describes the evolution of the vertical elevation of the water level compared to base line functions of fetch and of the wind speed.

The evolution of the period based on wind speed and fetch, changes in height depending on the wind speed and fetch and the evolution of the wavelength depending on the speed and the fetch are characterized respectively by figures 2c, 3c and 4c.
Figure 5: Variations of $z$ according to wind speed and fetch

Figure 2c: Variations of adult wave period according to wind speed and fetch

Figure 3c: Variations of adult wave height according to wind speed and fetch
ANALYSIS AND DISCUSSION

The wind is a major factor in the rise of waves; the results the following are obtained.

Diagram of figure 1a reflects changes in the speed of swell function of time and the wind speed at constant fetch. It shows that when the speed increases, the speed increases and its maximum value are quickly reached. Diagram of figure 1b shows the evolution of the speed versus time and fetch but speed Constant wind and we find that the change does not affect fetch the maximum value of the speed of the waves.

Diagram of figure 2a shows the variation of the period based on time and speed to fetch. This curve shows that as speed increases the maximum period is quickly reached. This confirms the observations made by Patrick (2001) in the fifth chapter of his thesis and those of Kabir (Sadeghi, 2007).

Figure No. 2b shows the evolution of the period with time and fetch at constant speed wind. It shows that the maximum period does not depend on fetch but the wind speed. This confirms the results Press experimental measurements (Kabir Sadeghi, 2007)

Diagram of figure 3a that shows the evolution of the wave function in length time constant fetch, we see that when the speed increases the maximum wavelength is quickly reached. Diagram in Figure 3b which reflects changes in the wavelength over time at constant wind speed, we deduce that the maximum wavelength does not depend on fetch but the wind speed. These observations are consistent with the model comparison made by: ROSALES-Victor SIERRA (2004).

Diagram of figure 4a shows the evolution of the wave height based on time constant fetch. We see that as the speed increases the wave height increases and its maximum value is quickly reached. This result is confirmed by the work of (Christian Seibt et al, 2013 and Kabir Sadeghi, 2007). Diagram of figure 4b reflects the height variation with time at constant speed wind. We see that as the fetch increases the wave height increases as well. Then the wave height depends on the fetch and wind speed.

Diagram of figure 5 shows the evolution of the vertical elevation of the water level compared to the reference level based on the fetch and speed vent. This is confirmed by the work JC HAMBAREL 2009

Figure 2c shows the evolution of the period depending on the speed and the fetch. It appears that the period changes linearly when the fetch is growing and when the speed increases also. Diagram of figure 3c shows the evolution of the height depending on the speed and the fetch. It shows that the height changes linearly as the speed increases and increases logarithmically when the fetch is growing. Of figure 4c that reflects changes in the wavelength depending on the speed and fetch, we deduce that the wavelength evolves exponentially when the fetch believes and when the speed increases also.

CONCLUSION

The analytical prediction model established shows that the variables that characterize a wave that is born in the wind are increasing functions in the wind speed, fetch and time. Apart from the speed, the period and the wavelength which become constant after duration $t_F = \frac{L}{U}$, height continues to grow that after a time. This model helps to model a wave from birth to age adult, leading to predictions of the characteristics of a wave those who know the wind. Apart from any other disturbance, the results contribute to the dynamics followed of the ocean on a given territory. We believe in then use this model to quantify energy waves, deduce the potential power that could be drawn on the coast of a country in after measuring speed high near sea wind country.

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